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EFFECT OF A GAS CAVITY ON A PRESSURE SURGE IN A HYDRAULIC LINE

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UDC 532.595.2+532.595.7

Situations in which gas-filled cavities are present in the fluid are encountered in the operation of various hydraulic systems. This is sometimes the result of the accidental admission of air into the line, while in other cases it is due to the presence of air chambers placed in the system to damp pulsations of the fluid.

It is known that the presence of a macroscopic volume of gas in a hydraulic line can sometimes appreciably intensify pressure fluctuations occurring during transients [1-4]. For example, during the filling of a pipeline with fluid, a hydraulic shock 10 times greater than the pressure of the feed tank is realized [2]. The author of [3] studied the hydraulic shock which occurred when a pipeline provided with an air chamber and filled with a viscous fluid was rapidly connected to a tank under constant pressure. It was found that when the relative volume of air $\alpha_V < 10^{-2}$, the presence of a chamber designed to damp pressure surges leads to some increase in maximum pressure (by 30%). Only at $\alpha_V > 3 \cdot 10^{-2}$ does the chamber alleviate hydraulic shocks.

A numerical method was used in [4] to study the effect of the gas cavity on the pressure maximum for the case of instantaneous opening of a valve with a low hydraulic resistance. The investigation established the optimum gas volume at which the hydraulic-shock-induced increase in pressure would be maximal. This value is several times greater than the maximum pressure in a pipeline without a gas cavity. If the volume of the gas cavity is large enough, it acts as a damper and lowers the maximum pressure. Thus, depending on the parameters of the hydraulic system, a localized gas volume can either relieve pressure from a hydraulic shock or increase the pressure to a level which is dangerous for the system.

It should be noted that the authors of [1-4] did not study the effect of the loading of a pipeline by pressure. However, this parameter is important because a slow "application" of the load (gradual opening of a valve, etc.) is the method usually employed to eliminate dangerous pressure surges during transients in hydraulic systems.

In the present study, we experimentally and theoretically examine a transient involving the loading of a pipeline with pressure when the line has a gas cavity at the end. In contrast to [3, 4], the characteristic period of pressure build-up at the inlet of the system corresponded to several traversals of the line by a wave. Thus, the hydraulic-shock character of the transient was fairly weak.

A diagram of the test unit is shown in Fig. 1. One end of a steel pipe 5 with a length L = 2.3 m and a diameter d = 22 mm was connected by means of an adapter 2 and electromagnetic valve 1 to an air main at a pressure $P_1 = 7 \cdot 10^5$ Pa. A steel cylinder 6 with a

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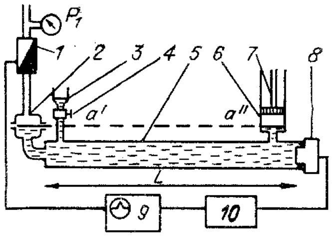

Fig. 1

piston 7 was connected to the other end of the pipe and a pressure gauge 8 was installed in the same end. The pipe was filled with water through the funnel 3 to the level $a'a''$. The cock 4 was then closed. The initial volume of air in the cavity was established by movement of the piston. Its position was then fixed. When the valve was activated, air entered the adapter from the main. This produced an increase in pressure at the inlet of the pipe. The S1-17 oscillograph 10 began to record when the valve was opened. The oscillograph recorded the signal from a DD-10 pressure transducer. The frequency characteristic of the transducer, powered by an IVP-2 block 9, was nearly linear up to 10 kHz (carrier frequency 30 kHz). We also measured the velocity of propagation of the compression wave along the pipe by means of a second pressure gauge positioned in the middle part of the pipe.

The measurement results were compared with a theoretical relation giving the dependence of the pressure in the end section of the pipe on time. In the calculations, the motion of the fluid was described by the equations for hydraulic shock [5]:

$$\frac{\partial P}{\partial t} + \rho c^2 \frac{\partial u}{\partial x} = 0, \quad \rho \frac{\partial u}{\partial t} + \frac{\partial P}{\partial x} + \frac{\lambda \rho u |u|}{2d} = 0. \tag{1}$$

Here, $P(x, t)$, $u(x, t)$ are the pressure and velocity of the fluid; ρ is density; c is the velocity of the compression wave in a fluid-filled pipe of diameter d; λ is the friction coefficient on the pipe wall.

The initial conditions at the moment of opening of the valve:

$$u(x, 0) = 0, \quad P(x, 0) = P_0, \quad P_0 = 10^5 \text{ Pa}. \tag{2}$$

In the initial section of the pipe, P and u are connected by the relation

$$x = 0: \; P = P_0 + \Delta P - \xi_1 \rho u |u|/2, \tag{3}$$

where ξ_1 is the hydraulic resistance at the pipe inlet; $\Delta P(t)$ is the increase in pressure in the adapter after opening of the valve. The relation $\Delta P(t)$ is determined by many parameters, including the volume of the adapter, the cross section of the valve, and the pressure in the main. Due to the complexity of accounting for all of these parameters, in our calculations we assigned the function $\Delta P(t)$ in the form

$$\Delta P(t) = P_1(1 - \exp(-t/\tau_1)) + P_2 \exp(-t/\tau_2) \sin \omega t. \tag{4}$$

The characteristic period of pressure build-up τ_1 was chosen from a comparison with the experimental results. The second term in (4) accounts for the oscillations which develop during the compression of the air in the adapter, since the adapter, together with the pipe leading to the valve, forms a Helmholtz resonator with the natural frequency $\omega = 1.05 \cdot 10^3$ $\sec^{-1}$. These vibrations, with a characteristic rise time τ_2, are clearly distinguishable on certain oscillograms (as a series of decaying "peaks" at $t < 15$ msec in Fig. 2b, for example) and have nothing to do with the presence of the gas cavity. They are extraneous in relation to the process being studied here. The coefficients P_1 and P_2 are constant and are determined from the conditions of the experiment.

The boundary conditions in the end of the pipe where the gas cavity is located

$$x = L: \; dV/dt = -\pi d^2 u/4, \quad P - P_g = \xi_2 \rho u |u|/2, \quad P_g V^{\varkappa} = P_0 V_0^{\varkappa}. \tag{5}$$

Here, P_g and V are the pressure and the volume of the gas under the piston (V_0 is the initial volume); P and u are the pressure and velocity of the fluid in the end of the pipe; ξ_2 is the hydraulic resistance between the pipe and the cylinder. Equation (5) means that the compression of the air in the cavity of the cylinder is assumed to be adiabatic, $\varkappa = 1.14$.

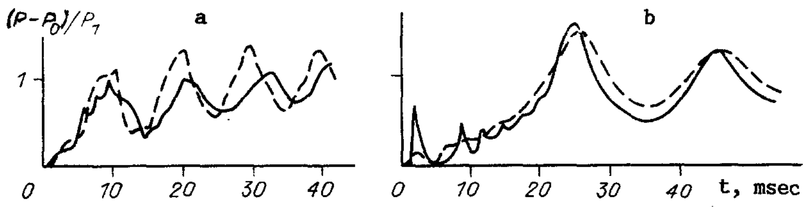

Fig. 2

Having excluded P_g and V from (5), we easily obtain a relation which connects the pressure and velocity of the fluid in this section.

Equation (1), with initial and boundary conditions (2)-(5), was solved by the method of characteristics. Here, the boundary conditions were augmented by the relations for the characteristics in the corresponding sections. We used the following parameter values in the calculations: c = 1170 m/sec (from measurement results); $P_1 = 5 \cdot 10^5$ Pa; τ_1 = 10 msec, τ_2 = 5 msec (chosen from a comparison with the experimental data); $\xi_1 = 450$, $\xi_2 = 500$ (determined by the cross-sectional areas of the adapter and the pipe).

Figure 2 shows oscillograms of the pressure-gauge signals (solid lines) in comparison with the calculated results (dashed lines) for two values of the relative volume $\alpha_V = 4V_0/\pi d^2 L$ of air in the cavity: $\alpha_V = 0.001$ (a) and 0.008 (b). It can be seen from the curves that pressure increases nonmonotonically, which is due to the oscillatory character of motion of the fluid in the pipe. In the case of a low content of gas (Fig. 2a), the time interval ΔT between successive pressure maxima (the period of oscillation) is only slightly greater than the quantity $4T_0$ ($T_0 = L/c$) representing the period of hydraulic-shock-induced oscillations in a pipe without a gas cavity. The maximum increase in pressure in this case barely exceeds P_1. The hydraulic-shock character of the process is weakly expressed due to the relatively slow increase in pressure $\Delta P(t)$ in the adapter. The small increase in the volume of air in the cavity leads to a substantial increase in both the period and the "amplitude" of the pressure oscillations. For example, the pressure increase in the first peak (Fig. 2b) is $1.65P_1$. This can be attributed to the fact that the presence of compliant gas volume in the end of the pipe leads to a large acceleration of the fluid under the influence of pressure $\Delta P(t)$ in the initial section of the pipe [4]. As the fluid subsequently slows, its kinetic energy is converted into the elastic energy of the compressed gas. This accounts for the pressure maxima in Fig. 2b. Thus, liquid in a pipe with a gas cavity constitutes an oscillatory system. Meanwhile, the mass of the liquid determines the inertia of the system, while the gas volume plays the role of a nonlinear elastic spring.

The period of oscillation ΔT increases with an increase in the volume of the cavity — as shown in Fig. 3 — since an increase in the size of the gas volume is accompanied by an increase in the compliance of the "gas spring." It can be seen that the experimental points (each point being the result of 5-7 measurements) agree quite well with the calculation. It should be noted that in the range we studied, $\alpha_V < 0.02$, the characteristic time of the oscillations is comparable to the time T_0 it takes the compression wave to travel along the pipe. As a result, the piston model of fluid motion in [4] is invalid in the present case.

The fluid vibrations which occur with loading of the system die out over time due to hydraulic losses, thus accounting for the smaller size of the maxima after the first pressure maximum (see Fig. 2b). As was shown by the calculations, the main contribution to attenuation of the vibrations is made by hydraulic resistance concentrated at the ends of the pipe. Friction losses on the pipe wall have almost no effect on the amplitude of pressure.

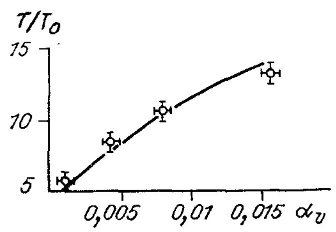

Fig. 3

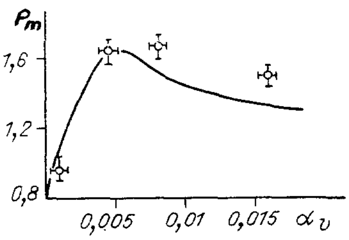

Fig. 4

Figure 4 shows the dependence of the size of the first pressure maximum on the gas volume in the dimensionless variables $P_m = P_m(\alpha_V)$, $P_m = P_{max}/P_1$, where P_{max} is the pressure in the first peak. The initial increase in pressure with an increase in α_V – due to intensification of the effect of the gas cavity on the acceleration of the liquid – is subsequently replaced by a smooth decrease in α_V due to the increasing role of hydraulic losses [4]. At $\alpha_V > 0.007$, the experimental data is somewhat higher than the theoretical results. This may be connected with the fact that, in the calculations, hydraulic losses were accounted for by means of constant coefficients ξ_1 and ξ_2 taken for a steady flow. For nonsteady motion, however, hydraulic resistance may depend on the instantaneous values of fluid velocity and acceleration.

Thus, in the investigated case of relatively slow loading of a hydraulic system, the presence of a small volume of gas ($\alpha_V \sim 0.01$) leads to a substantial (albeit less than for instantaneous loading) increase in the pressures realized in the transient. The possibly dangerous effect of a localized gas volume such as that examined here should be considered in the analysis of transients in hydropneumatic systems.

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WAVE FLOWS OF A CONDUCTING VISCOUS FLUID FILM IN A TRANSVERSE MAGNETIC FIELD

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Investigation of the wave regimes occurring in thin layers of a viscous weak-conducting fluid in magnetic and electrical fields is of interest in connection with the prospective utilization of film flows in nuclear power [1] and other technological processes. Experimental and theoretical investigations of wave effects in structures that occur on the free surface of an ordinary (non-electrically conducting) viscous fluid showed that these phenomena influence the stability and evolution of the film flows substantially [2-4]. The theory of the wave motion of a laminar viscous film surface was first developed by Kapitsa [2]. The critical value of the Reynolds number was obtained for which a wave mode is built up in the film when it is exceeded. It is shown that the mass transfer is improved in films in the wave mode as compared with ordinary flow conditions. At this time magnetohydrodynamic flows of conducting viscous fluid films are studied intensively [5-7]. A mathematical model is proposed in [5] for a flow with a free surface of the liquid–metal diaphragm of a power plant. The asymptotic of the surface of the spreading film in transverse electrical and magnetic fields is presented in [6]. The stability of a laminar flow of an electrically conducting fluid film is considered in an induction-free approximation in [7] on the basis of the Orr–Sommerfeld equation.

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